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RELATIVISTIC SCALING OF PHYSICAL PROPERTIES: RECIPROCAL RELATIONSHIP BETWEEN CONVERSION FACTORS

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ABSTRACT

It is well established experimentally that clock rates increase with gravitational potential. This effect was predicted by Einstein in 1907 and verified in the experiments of Pound and Snider in 1965. In the present work it will be argued that the above results can be conveniently described in terms of a uniform scaling of physical properties. One only needs a conversion factor between the two sets of units in different gravitational potentials to accurately predict the result of a measurement at one altitude based on the corresponding value obtained at another. For the case of the unit of energy, Einstein showed that the appropriate conversion factor is $S = 1+ghc^{-2}$, where c=2.99792458 ms-1 is the speed of light in free space, g is the local acceleration of gravity and h is the difference in altitudes between the two rest frames. The corresponding factors for light speed and frequency are both also equal to S. As with conventional measurements, the corresponding conversion factor in the reverse direction is always the reciprocal of the other. For example, the reverse factors for energy, light speed and frequency are each equal to $S^{-1} = 1-ghc^{-2}$. Attempts to develop a corresponding set of conversion factors for different inertial rest frames have heretofore been hampered by the fact that the Special Theory of Relativity (STR) predicts unambiguously that time dilation is symmetric, i.e. that a moving clock is always found to have a slower rate than one that is stationary in the observer's rest frame. On this basis, it is impossible to define a unique conversion factor between measured values of the same frequency obtained in two different rest frames. The present work shows that experimental tests of the symmetry of time dilation do not agree with the above prediction of STR. As a result, it becomes possible to also define conversion factors between measured values in different inertial systems.

KEYWORDS: *Einstein's Symmetry Principle (ESP), Asymmetric Time Dilation, Clock-Rate Proportionality, Universal Time-Dilation Law (UTDL), Lorentz Transformation (LT), Relativistic Velocity Transformation (RVT), Alternative Global Positioning System-Lorentz Transformation (GPS-LT), Absolute Remote Simultaneity, Isotropic Length Expansion, Uniform Scaling of Physical Properties, Amended Version of the Relativity Principle (RP)*

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INTRODUCTION

The classical treatment of gravitational interactions is based on the energy conservation law. Accordingly, there are two distinct types of energy, potential and kinetic, that are exchanged when a body is in free fall. Newton was able to predict quantitative details of planetary orbits on this basis, and his theory is still used exclusively in modern-day navigation systems. After introduction of his Special Theory of Relativity (STR) [1], Einstein derived relationships between measured values of properties for the same system obtained by two observers located at different positions in a gravitational field [2].

He predicted that the rates of clocks increase with gravitational potential [3], for example.

Because of a locality principle, however, Einstein assumed that the *in situ* frequency of an atom will always be the same, i.e. be independent of the location of the observer and his stationary clock. Formally, one can attribute the distinctions between the measured results for the same quantity obtained by observers located at different gravitational potentials to the fact that they employ different sets of rational units to express their respective numerical values. He argued, for example, that the Maxwell equations "are of the same form" in different positions in a gravitational field, but that the value of the speed of light changes from one position to another. This conclusion led him to predict that light waves are bent in a gravitational field and ultimately led him to develop the general theory of relativity [4].

This experience suggests that it would be quite useful to be able to predict "conversion factors" between the units employed by observers located at different positions in a gravitational field. Einstein made definite predictions for the values of these quantities for light speed and frequencies, and experiment has subsequently verified his conclusions to a high degree of accuracy, as will be discussed in detail below. Particular emphasis will be placed on the possible existence of corresponding conversion factors for the units employed by observers in different states of motion.

Gravitational Scaling of Physical Units

The first example of a gravitational scale factor was given by Einstein [2] for light frequencies. He concluded that a clock at a higher gravitational potential Φ must run (1 + Φc-2) times faster than an identical clock located at the observer's location. He based his argument on considerations of space-time relationships expected from STR [1] and the equivalence principle (EP) [2]. The same value for the conversion factor for energy E can be obtained directly from Newton's gravitational theory and the mass-energy equivalence relation of STR. Accordingly, one assumes that the increase in energy of an object of mass m when it is raised to altitude h above the observer's location in the gravitational field will have a value of mgh, where g is the local acceleration due to gravity. The initial value of the energy of the object is equal to mc^2, where c is the speed of light in free space, so the fractional increase in energy is equal to the ratio of mgh to mc^2, which upon cancellation of m becomes $ghc^{-2}=\Phi\ c^{-2}$ [4, 5] (note that the object's gravitational mass m_G in the former quantity and inertial mass m_I in the latter have the same values in this example because no change in the object's state of motion is assumed when it is moved to a different gravitational potential). The ratio of the energy E(P) measured by the observer when the object is located at position P in the gravitational field to the corresponding value for the identical object when it is located at position O is $(1 + \Phi c^{-2}) = (1 + ghc^{-2})$, i.e.

$$E\ (O) = (1 + ghc^{-2})\ E\ (P) = S\ E\ (P). \qquad (1)$$

The corresponding relationship for light frequencies ν is [2]:

$$\nu\ (O) = (1 + ghc^{-2})\ \nu\ (P) = S\ \nu\ (P). \qquad (2)$$

Moreover, Einstein obtained a similar relationship for measured values of the speed of light [2]:

$$c\ (O) = (1 + ghc^{-2})\ c\ (P) = S\ c\ (P). \qquad (3)$$

As discussed above, it is assumed that a local observer at point P obtains the same values for the various quantities in eqs. (1-3) when the object is located there as the observer at O finds when the object is located there. This "locality principle" is essential for understanding the rationale of the scaling arguments. It is assumed that the reason the two observers measure different values for each of these quantities when the object is located at point P is because they employ different units to express their numerical values. Each observer thinks that he is using standard units in each case because all his *in situ* measurements are perfectly consistent with this assumption. In order to predict the values obtained by the observer at point O in the gravitational field to the corresponding values obtained locally at point P, it is necessary to recognize that the two sets of standard units are actually not the same. The conversion from one set of units to another is accomplished by employing the factor S in eqs. (1-3) in each case. The scaling is always completely uniform, so the same conversion factor is applicable for all quantities of the same type.

The situation is completely analogous to the procedure when one changes the unit employed in a given laboratory from feet to meters in distance measurements. One simply has to know the appropriate conversion factor to express the result in a different system of units. In absolute terms, the result is exactly the same no matter which set of units is employed. The problem with gravitational scaling is that each observer thinks that he is using the standard unit to express his results for any given quantity. To be completely accurate, one should not only mention the unit in each case but also the location in the gravitational field in which the property is measured. The point of the present exercise is that the conversion factors exist, not to explain why they exist. One can look upon such relationships as "natural laws" that are universally reliable.

Another feature of the analogy with unit conversions also needs to be emphasized in the present context, namely that the "reverse" conversion is always accomplished with the reciprocal of the conversion factor in the original direction. This fact is obvious from algebraic manipulation of eqs. (1-3). In order to convert the values obtained by the observer located at point O to those obtained by the observer at point P, it is necessary to use the factor S^{-1} in each case.

The overriding assumption in the above arguments is that measurement is completely objective. Two observers always experience the same event when carrying out their respective measurements. The only reason they can legitimately obtain different results for the same quantity is because they employ a different standard unit in which to express their numerical values. There is no absolute standard with which to compare, so the result of every measurement must always be given relative to a specific reference value. The *ratios* of numerical values for two measurements of the same quantity must be the same, however. The same holds true for ordinary measurements carried out at the same position in a gravitational field when the two observers employ different standard units such as feet and meters or seconds and nanoseconds. One can refer to this state of affairs as the principle of rational measurement (PRM) [7].

One of the main consequences of objectivity is that it allows one to use the ordinary rules of algebra to deduce other gravitational conversion factors than those given in eqs. (1-3). For example, the unit of frequency is s^{-1}, so it follows that the conversion factor for periods and other elapsed times is the reciprocal of that for frequencies, namely S^{-1}. The information for speed, energy and frequency is sufficient to derive the conversion factors for all physical quantities. The value for wavelengths/lengths is determined as the product of the conversion factors for speed and time and is therefore seen to be unity (S^0). Similarly, the conversion factor for momentum p can be obtained from the E=pc formula of quantum mechanics. Once again the result is unity. The conversion factor for angular momentum, with unit Js., is the same as for Planck's constant h and also has a value of S^0. In other words, observers located at different gravitational potentials should agree on the values of each of these three quantities. It must be emphasized, however, that this conclusion only holds if the object of the measurement is stationary with respect to each of the observers. This is the general condition for application of the gravitational scale factors. Finally, the conversion factor for inertial mass m is deduced to be S^{-1} from the fact that this quantity is the ratio of momentum p to speed v. The result is seen to be the same as for time. The conversion factor for angular momentum (mvr) can thus be computed as the product of the corresponding factors for inertial mass (S^{-1}), speed (S) and distance (S^0), consistent with the result obtained above (S^0) based on the conversion factors for energy and time. More details on the general theory of gravitational scaling of physical units may be found in the author's earlier publications [8-10].

The conversion factors in eqs. (1-3) are only applicable in small regions of space where the value of g can be considered to be constant. A general value can be obtained by integration of the fractional increase in energy from a given point r_i in the gravitational field up to infinity and using Newton's universal law of gravitation to define g at each interval ($G = 6.670 \times 10^{-11}$ Nm^2/kg^2 is the universal gravitational constant and Ms is the active mass responsible for the interaction):

$$\text{Int } r_i^{\infty} (gc^{-2}dr) = \text{Int } r_i^{\infty} (GM_s r^{-2} c^{-2} dr) = GM_s r_i^{-1} c^{-2}. \quad (4)$$

Note that the value required for light speed is the *local* value at each infinitesimal interval and can therefore be set equal to c throughout, i.e. can be treated rigorously as a constant independent of r in the integrand. It is then useful to define the conversion factor A_i between the unit of energy at position r_i in the gravitational field and the corresponding value at infinity:

$$A_i = 1 + GM_s r_i^{-1} c^{-2}, \quad (5)$$

a value of the energy obtained at infinity must be multiplied by A_i to obtain the corresponding result for the observer located at position r_i in the field.

The next step is to obtain the conversion factor between the observer's unit of energy and that employed by an observer at the same location r_p in the field as the object of a given measurement. This can be done by first computing the reverse conversion factors between the unit at infinity and those for the observer's location at r_o and the object's location at r_p; these two results are A_o^{-1} and A_p^{-1}, respectively. The conversion factor S to be used by the observer at r_o for the object at r_p is then obtained by division as:

$$S = A_o/A_p. \quad (6)$$

Note that this value of S is consistent with that appearing in eqs. (1-3) if we set $h=r_p-r_o$ for small/infinitesimal values of h. Einstein noted in his original work [2] that the value of (S-1) on earth for an object located at the sun's chromosphere is 2.1×10^{-6}. It is also clear from eq. (6) that the reverse conversion factor is A_p/A_o, the reciprocal of S, consistent with the requirement discussed above.

It is impractical to carry out the experiment with solar light frequencies because of the random motion of the atoms located near the sun, but a terrestrial experiment [11, 12] has confirmed eq. (1). The Mössbauer effect was used to measure the frequency of x-rays emitted from a source located 22.5 m above the detector. The observed frequency agreed to within experimental error with the expected value of Sν, where ν is the frequency measured at the light source. The interpretation dating back to Einstein's original work [2] is that the unit of frequency is S times smaller at the location of the x-ray absorber, and that this frequency is maintained as the radiation descends to lower altitude. The effect of the gravitational scaling on other properties has been discussed elsewhere [13-14] and will be briefly reviewed in the present work. The value of the energy measured at the location of the absorber is Shν = SE and the corresponding speed of light is Sc. The energy of the radiation is also constant during free fall, but the light speed decreases to a value of c at the absorber location. On this basis, one can conclude that the wavelength of the radiation decreased by a factor of S^{-1} from the initial value of λ, whereas the momentum increased to a value of SE/c from its initial value of p =E/c. The product of wavelength and momentum therefore remains constant, in agreement with the de Broglie relation, $p=h/\lambda$ under the assumption that Planck's constant h is constant throughout, all of which is consistent with the previous discussion in this section.

One of the main theoretical goals of the original experiment was to measure the effective weight of photons. The latter quantity is equal to $gh\nu/c^2$ at the location of the x-ray source. As discussed above, inertial mass scales as S^{-1}, so in the units of the observer at the ground level, it has a value of $S^{-1}\, h\nu/c^2$. At the conclusion of the experiment, the speed of light has decreased by a factor c, whereas both the frequency ($S\nu$) and h are unchanged. Therefore, the inertial mass of the photons has increased by a factor of S^2 to a value of $S\nu/c^2$. Since g has essentially the same value throughout, it therefore can be concluded that the weight of the photons has also increased by a factor of S^2 by the time they reach the absorber.

A more straightforward application of eq. (1) involving atomic clocks has also been reported [15]. In agreement with expectations, the rate of the clock at a higher altitude was found to be S times larger than that of its identical counterpart at ground level.

Direct tests of the effect of gravitation on the speed of light are also impractical, but they can be deduced on the basis of certain experiments. For example, Shapiro [16] has shown that the speed of radio signals decreases when they pass close to planets such as Mercury and Jupiter [17]. The strongest evidence for the gravitational scaling of light speed comes from numerical calculations that predict the displacement of star images during solar eclipses. Quantitative agreement with the angle of bending of wave fronts was obtained by Einstein [4] using his Theory of General Relativity. Schiff [18] has shown that this angle can be computed to the same level of accuracy in a simplified treatment based entirely on scaling arguments. It was necessary for him to take special account of the fact that the conversion factor changes continuously as the light travels radial to the gravitational field of the sun [10, 13, 18-20].

It is probably also fair to say that the scaling of energy has been shown to be accurate based on indirect experience with the other properties. The derivation of the conversion factor in eq. (1) relies solely on the formula for potential energy in Newton's classical theory and comes to the same result as for frequencies and light speed than was obtained by Einstein using a different route. As already discussed, once the conversion factors for these three quantities have been determined, one only has to rely on a belief in the fundamental objectivity of measurement to derive the corresponding value for any other physical quantity based on its composition in terms of these fundamental units.

Another advantage of objectivity is that allows one to easily convert the scale factors of one observer to the corresponding values of another. One simply has to know the ratio X of the new observer's A_i value from eq. (5) at location r_i in the gravitational field to that of the original observer, i.e. $X= A_i/A_o$. It is clear from eq. (6) that the product of X with the scale factor $S_{old} = A_o/A_p$ for the observer at r_o accomplishes the desired conversion: $S_{new}=X\ S_{old} = (A_i/A_o)(A_o/A_p)=A_i/A_p$.

Thus far in the discussion, it has been assumed that there is only one active mass that needs to be considered in computing gravitational scale factors. In principle, of course, clock rates and other properties could be influenced by a number of such masses. The notation used above can be changed to account for this eventuality by adding a superscript m to the A_i quantities defined in eq. (5), i.e. A_i^m, to distinguish between the various possibilities. The corresponding conversion factors for multiple active masses would then be computed as *products* of the individual factors for the various masses, m=1,2...n. Consequently, the presence of a second body m can either increase or decrease the value of the conversion factor from eq. (6) obtained for just the main active mass depending on whether A_o^m is greater or less than A_p^m.

Kinetic Scaling of Physical Units

The previous discussion of gravitational scale factors raises the question of whether a similar program for comparing experimental results can be constructed for pairs of observers in different rest frames. It should be clearly recognized that this **cannot** be done as long as one adheres to Einstein's STR [1]. The reason is because STR conforms to a symmetry principle that implies that the measurement process is subjective in nature. Which of two identical clocks is running slower is purely a matter of perspective in this theory, for example. One can only have a workable system of units if it is possible in principle to always know not only which clock is faster, but also by what fraction.

There is reason to believe that STR [1] is wrong on this point, however. For example, the belief that the results of measurements depend on the speed of the object relative to the observer has a significant effect on the way conservation of energy principle is to be applied. Consider the case of an object falling through a distance h. An observer for which the object is initially at rest will find that it as speed v when it reaches the end of its motion downward. Accordingly, the kinetic energy at this point has changed from a null initial value to $mv^2/2$. This value exactly cancels the loss in potential energy (mgh), so energy is conserved from this observer's vantage point. The situation is different for another observer traveling at a constant downward speed v. He finds that the initial speed of the object is v in the upward direction, whereas it attains a null value at the end of the fall. In this case the kinetic energy has *decreased* by the same amount as for the first observer, while the change in potential energy has the same value. Therefore, energy is not conserved from the vantage point of this observer. If conservation of energy is believed to be absolute, this dependence on the state of motion of the observer relative to the object is totally unacceptable.

It is important to realize that the symmetry principle of STR has remained unconfirmed based on experiment. On the contrary, there are numerous examples where it is violated. The first such definitive test was carried out by Hay et al. [21] in which an x-ray source and absorber were both mounted on a high-speed rotor. The authors found that the frequency shift Δν of the x-rays measured at the absorber is described by the following empirical formula:

$$\frac{\Delta\nu}{\nu} = \left(R_a^{\ 2} - R_s^{\ 2}\right)\frac{\omega^2}{2c^2}, \tag{7}$$

In which ω is the circular frequency of the rotor and R_a, R_s are the radial distances of the absorber and the light source, respectively. The same formula was subsequently verified by two other groups [22,23]. It is obvious from eq. (7) that the emission process does not satisfy the STR symmetry principle. The results indicate that the sign of the shift differs depending on whether the absorber is closer to the axis of the rotor or farther away from it. The STR symmetry principle would have us believe on the contrary that Δν would depend on the absolute value of the difference of the two radii, as shown below in eq. (8):

$$\Delta\nu/\nu=-|R_a-R_s|^2\omega^2/2c^2, \tag{8}$$

so that a decrease in frequency (red shift) would be observed in all cases. Instead, a blue shift is measured when the absorber is mounted at the rim of the rotor, i.e. with $R_a > R_s$. Kündig [23] summarized this result quite succinctly as follows: the clock undergoing greater acceleration runs slower. This statement is tantamount to saying that the frequency measurements are objective after all.

As a result, it is possible to use eq. (7) to derive a scaling factor Q for kinetic effects that is wholly analogous to the gravitational scaling factor S discussed in the previous section. If the frequency measured at the absorber is defined as

$\nu_a=\nu_s+\Delta\nu$, eq. (8) can be reformulated as:

$\nu_a/\nu_s = 1 + (R_a^2-R_s^2)\omega^2/2c^2$, (9)

which in turn can be seen as an approximation to the more general form involving γ factors shown below:

$\nu_a/\nu_s = \gamma(R_a\omega)/\gamma(R_s\omega)= \gamma(v_a)/\gamma(v_s)$, (10)

v_a and v_s are the speeds of the absorber and x-ray source relative to the rotor axis. If we then associate the absorber with observer O in eqs. (1-3) and the light source with position P, eq. (10) can be rewritten similarly to eq. (2) as:

$\nu (O) = [\gamma(v_o)/\gamma(v_p)]\, \nu (P)$. (11)

The corresponding formula for the periods of the radiation T =1/ν is then obtained as:

$T (O) = [\gamma(v_p)/\gamma(v_o)]\, T (P) = Q\, T (P)$, (12)

Where by

$Q= \gamma(v_p)/\gamma(v_o)$ (13)

Can be looked upon, analogous to S in eqs. (1-3), as the conversion factor between the units of time in the two rest frames.

The above analysis is clearly supported by the experiment carried out a decade later with circumnavigating clocks by Hafele and Keating [24-25]. They found that the elapsed times recorded on atomic clocks were inversely proportional to γ (v), where v is the speed relative the center of the earth (ECM). As a consequence, clocks flying westward ran at slower rate than those at the airport of departure, while the latter in turn ran slower than the clocks flying in the westerly direction. These results could be quantitatively explained by the fact that the earth was rotating eastward below the clocks, thereby increasing the effective speed of the clocks moving in that direction and decreasing it in the opposite case. Both the rotor and airplane data are thus seen to satisfy the following general formula for elapsed times Δt and Δt':

$$\Delta t\, \gamma\left(v_0\right) = \Delta t'\, \gamma\left(v_0'\right). \quad (14)$$

The corresponding speeds v_0 and v_0' are measured relative to a specific reference frame and generally not relative to that of the observer. The latter rest frame is the rotor axis in the Hay et al. study [21-23] and the ECM in the airplane experiment [24, 25]. In previous work, this reference has been referred to as the objective rest system (ORS) [26]. In Einstein's example [1] of an electron moving in a closed circle, the ORS is the point at which force is applied to the electron to cause it to be accelerated. For the case of an object in free fall discussed in the preceding section, the ORS is the rest frame in which the object is initially stationary. The state of motion of the observer therefore has no effect on the calculation of the total energy at different stages of the free fall, with the result that the conservation of energy principle holds throughout.

Because eq. (14) describes the variation of the elapsed times of clocks quantitatively in all these cases, it is appropriate to refer to it [27] as the Universal Time-Dilation Law (UTDL). It is seen to be compatible with eqs. (12, 13) and thus with the general conclusions regarding the kinetic scaling of time and frequency, whereby $Q=\gamma(v_0')/\gamma(v_0)$ in this notation. In order to apply the UTDL in a given case, it is first necessary to identify the ORS. The *conversion factor* Q between the units of time in different pairs of rest frames can then be computed directly just by knowing their respective speeds relative to the ORS. Unlike the case for the STR treatment of time dilation (Einstein's symmetry principle [1]), the relative speed of the latter two rest frames is not required for determining the ratio of their measured elapsed times. The key point is that the two observers in these rest frames must agree on the *absolute* value of the elapsed time after accounting for the difference in the units in which each expresses his measured result. As with the gravitational scaling discussed first, measurement is perfectly objective in this view. It is not a matter of the perspective of the observer.

Experiment allows us to deduce the kinetic scale factors for all other physical quantities. To begin with, the constancy of the speed of light implies that the conversion factor for velocity is unity (Q^0). This result only applies to the *relative* velocities of two objects. Clearly, the speed of a given object relative to two observers in different rest frames is not the same for both. The conversion factor for wavelength and distances in general must be the same as for elapsed times (Q). This is because a given distance L can be determined by measuring the elapsed time ΔT for a light pulse to travel between the two end-points. Light-speed constancy therefore implies that the value of L is equal to cΔT. Therefore, L and ΔT must change in the same proportion when the units of two observers are exchanged, since they must agree on the value of c. This means that isotropic length expansion must accompany time dilation in a give rest frame, i.e. when clocks slow down, both the unit of length and time should *increase* by the same fraction. This prediction stands in distinct contrast with the FitzGerald length contraction expected to accompany time dilation according to STR [1]. Experimental verification for the increase in length with increasing speed v was already obtained in 1938 with the transverse Doppler study of Ives and Stilwell [28]. The found that the wavelength of light *increases* by a factor of γ (v) in an accelerated rest frame (after averaging out the effect of the to-and-fro motion of the light source relative to the laboratory), and this result has also been obtained in subsequent tests of higher accuracy [29] than the original.

Experimental studies of the variation of inertial mass m with speed v [30] have indicated that it is also directly proportional to γ (v), the same as for time, so that the conversion factor for inertial mass must also be equal to Q. Since c has a constant value for all observers (at the same gravitational potential), the conversion factor for energy E is deduced from Einstein's $E=mc^2$ relation [1] to also have a value of Q. The value for angular momentum l=mvr can be determined on the basis of the above conversion factors as Q^2. The same result holds for Planck's constant h, since it has the unit of angular momentum (Js). This conclusion allows one to verify that the radiation law, E=hν, holds for all observers independent of their state of motion since ν varies as Q^{-1}, the reciprocal of the value for time, while E varies as Q. The corresponding factor for momentum p=mv is also Q, whereby once again it is assumed that the speed v is measured relative to the appropriate ORS in each case. As in the case of energy, this conclusion allows one to conclude that momentum is conserved for all observers, not just for someone who is stationary at the point of a given collision.

Consistent application of the above rules shows that the conversion factor for force F is Q^0 since it is the ratio of energy to distance. The factor for acceleration a can then be obtained in two different ways, namely in terms of the definition $a=d^2r/dt^2$ and also the F=ma relation of classical physics. In both cases the result is Q^{-1}. The same result holds for the acceleration due to gravity g. Newton's Universal Gravitational Constant G has units of Nm^2/kg^2 and therefore scales as Q^0. The acceleration g is defined as GM/r^2 for active mass M and radial distance r, which again leads to the value of the conversion factor of Q^{-1}. Note that in this computation no distinction is made between inertial and gravitational mass, since both have the unit of kg. This allows for a consistent choice for the conversion factor for gravitational potential energy, E=mgh, where m is the active mass in the calculation. In this case E also is found to have a conversion factor of Q^1. The exponent of +1 is obtained as the sum of the corresponding three factors for m (+1), g (-1) and h (+1).

The UTDL of eq. (14) also allows one to change units from one observer (O) to another (X). The Q value for O needs to be multiplied by $\gamma(v_O)/\gamma(v_X)$ to obtain the corresponding value for X, where v_O and v_X are their respective speeds relative to the ORS.

Thus far in the discussion it has been tacitly assumed that the ORS is the same for both the object and observer, as is assumed in the UTDL. When that is not the case, for example if the object is in the gravitational field of the moon while the observer is located on the earth's surface, it is necessary to know the ratio of the unit of time in the two ORSs. Since the moon orbits around the earth, clocks at the moon's center of mass will run slower than those at the ECM by a factor of $\gamma(v_M)$, where v_M is the orbital speed. In that case the initial value of Q for the observer computed as if the object were in the same gravitational field must be *multiplied* by $\gamma(v_M)$ to obtain the desired conversion factor. If the roles are reversed and the object is near the earth while the observer is in the moon's gravitational field, the initial Q value must be divided by $\gamma(v_M)$ because now the observer's rest frame has the slower clocks.

New Lorentz Transformation

It is important to recall from the beginning of this section that the entire kinetic scaling procedure is not consistent with the STR assumption of symmetric time dilation. The latter assumption is derived [1] from the Lorentz transformation (LT) and its condition of Lorentz invariance. In the past, physicists [31] have explained the fact that experiment always finds that clock rates are proportional, and therefore that time dilation is asymmetric, by claiming that the LT is not applicable in situations where either the object or the observer is under the influence of an unbalanced force. This claim is simply a euphemism to avoid admitting that the LT is contradicted by experiment in all these cases. Moreover, there has never been an experimental verification of symmetric time dilation. Added to this is the experience with high-speed electrons at CERN [32], in which it has been found that the degree of acceleration has no effect on the timing results. By extrapolation, this means that reducing the amount of acceleration to zero should not have any effect as long as the speed of the object relative to the laboratory (or other ORS [26]) remains unchanged.

There is an alternative means of making relativity compatible with Newton's First Law, however, namely to find another Lorentz-type transformation which is consistent with clock rate proportionality while still satisfying both of Einstein's relativity postulates. Such a transformation exists (referred to as the ALT or GPS-LT [33, 34]), and is shown below:

$$\Delta t' = \Delta t/Q \tag{15a}$$

$$\Delta x' = \eta\,(\Delta x - v\Delta t) \tag{15b}$$

$$\Delta y' = \eta\Delta y/\gamma Q \tag{15c}$$

$$\Delta z' = \eta\Delta z/\gamma Q, \tag{15d}$$

with $\eta=(1-vc^{-2}\,\Delta x/\Delta t)^{-1}$.

The condition of clock-rate proportionality is satisfied by eq. (15a) for the elapsed times Δt and $\Delta t'$ of the two observers. The ratio of clock rates Q expected from Newton's Law of Inertia is included explicitly in this equation as well as in the other three space relations. It is easy to see that these equations are consistent with Einstein's relativistic velocity transformation (RVT) by simply dividing each of the spatial variables $\Delta x'$, $\Delta y'$ and $\Delta z'$ by $\Delta t'$. It is therefore clear that the GPS-LT satisfies the light-speed constancy postulate. The condition for satisfying the relativity principle (RP) is that the inverse of these equations is obtained by reversing the sign of v and interchanging the primed and unprimed symbols in each case. It is seen from eqs. (15c,d) that this requires that $\eta\eta'=\gamma^2$ [with $\eta'=(1+vc^{-2}\,\Delta x'/\Delta t')^{-1}$ in accord with the definition of η given above] and QQ'=1. The latter condition simply implies that the conversion factors in the two directions have a reciprocal relationship, which is the normal situation in any kind of scaling procedure and is easily satisfied using eq. (13), namely with $Q'=\gamma(v_0)/\gamma(v_0')$. The satisfaction of the other condition can be proven with the help of the RVT [33,34].

It must be emphasized that the GPS-LT, like the LT before it, has been derived by assuming that both rest frames S and S' are inertial. The difference is that the GPS-LT makes the transition smoothly between timing results obtained when the observer and object are both moving freely to one in which one or the other of them is suddenly subjected to a slight force. In other words, there is no reason to expect that the clock-rate proportionality assumed in the GPS-LT is not retained when even a large force is applied. By contrast, textbooks espousing the symmetric time dilation predicted by the LT must assume that some very strange things happen when a similar transition occurs. Consider, for example, the case of a rocket moving at speed close to c as it passes over the earth with $\gamma(v)=1000$. According to the LT, an observer on the rocket should find that clocks on the earth run 1000 times slower as his from his vantage point when he is moving at constant velocity. Yet, the situation should change dramatically when the rocket undergoes a slight acceleration. Suddenly, the clocks on earth will be found to run 1000 times *faster* if Sherwin's conclusion [33] from the rotor experiment [21-23] is to be believed. It's fair to say that this state of affairs is totally unrealistic, even by the standards normally applied in discussing the wonders of relativity theory.

The GPS-LT of eqs. (15a-d) does not satisfy the condition of Lorentz invariance. However, it is consistent with a similar relationship that is obtained by summing the squares of each of its four equations, namely:

$$\varepsilon' (x'^2 + y'^2 + z'^2 - c^2t'^2) = \varepsilon (x^2 + y^2 + z^2 - c^2t^2), \tag{16}$$

where $\varepsilon=\eta/Q\gamma$ and $\varepsilon'=\eta'/Q'\gamma$. In order to satisfy the RP, it is necessary that $\varepsilon\varepsilon'=1$, and it has already been noted that this is the case for the latter values of ε and ε'. The LT, on the other hand, also satisfies eq. (16) by choosing a corresponding value of $\varepsilon=\varepsilon'=1$. This choice is obviously also consistent with the $\varepsilon\varepsilon'=1$ condition for satisfying the RP, but in addition, eq. (16) becomes the condition of Lorentz invariance with it:

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2 \quad . \tag{17}$$

It needs to be recognized that symmetric time dilation is a direct consequence of eq. (17), which, as discussed at length in Sect. II, has never been observed experimentally and is in fact contradicted by the high-speed rotor experiments [21-23] and the Hafele-Keating study with circumnavigating clocks [24, 25]. Lorentz invariance rules out clock-rate proportionality, and therefore is *also inconsistent with Newton's Law of Inertia*. In the past, as already mentioned, physicists have simply explained away the failure of the LT to correctly predict the asymmetry of time dilation in the above experiments by claiming [31] that the conditions for satisfying it, namely that both participating rest frames are inertial systems, are not satisfied in either case. There is no need for such *ad hoc* after-the-fact argumentation in the case of the GPS-LT. It predicts, or rather more accurately, postulates, that clock-rate proportionality will be observed even when the rest frames are both strictly inertial. What has not been recognized previously by the physics community as a whole is that the GPS-LT also satisfies Einstein's two postulates of relativity, but avoids the Lorentz invariance condition of eq. (17) by replacing it with eq. (16).

Description of Objects in Free Fall in Terms of Conversion Factors

Thus far in the discussion, the conversion factors for motion (Q) and gravity (S) between the various units employed by different observers have been considered separately. The question therefore arises as how to deal with situations in which the object of the measurement is located at a different gravitational potential than that of the observer as well as being in relative motion to him. By analogy to the conventional use of conversion factors, it seems reasonable that one should simply multiply the individual values for a given property to obtain the overall factor. This procedure has been followed for the three fundamental quantities of distance, inertial mass and time below (the corresponding units in the mks system are listed in parentheses in each case):

Distance (m) Q

Inertial mass (kg) QS^{-1}

Time (s) QS^{-1}.

The corresponding combined conversion factors for other properties can then be easily obtained on the basis of their composition in terms of the above three properties. Some key examples are listed below:

Energy (kgm^2s^{-2}, J) QS

Frequency (s^{-1}) $Q^{-1}S$

Velocity, c (ms^{-1}) S

Momentum ($kgms^{-1}$) Q

Angular Momentum, h (Js) Q^2.

Acceleration, g (ms^{-2}) $Q^{-1}S^2$

Force ($kgms^{-2}$, N) S

The first thing to notice is that these factors are consistent with numerous physical laws, including those from quantum mechanics [9, 10]. For example, the E=hν radiation law scales as QS on the left-hand side and as $(Q^2)(SQ^{-1})$=QS on the right-hand side. Thus the law holds for any relationship between the observer and the light source, as already discussed in Sect. III. The E=pc relation again scales as QS on the left for energy, while on the right, one has Q for momentum p and S for the speed of light c. The basic equation for gravitational energy, E=mgh, also holds, with the product $m(QS^{-1})g(Q^{-1}S^2)h(Q)$ again giving the conversion factor for energy E. The mass-energy equivalence relation has QS^{-1} for mass and S^2 for c^2. The formula for the phase velocity of light, νγ=c, also holds with frequency ν scaling as SQ^{-1} and wavelength as Q, giving a result of S for the light-speed conversion factor.

It is interesting to compare the results using the conversion factors with those expected from Einstein's equivalence principle [2]. The conventional view is that an upward acceleration of -g in an elevator in a gravitational free region of space is exactly equivalent to the effect of gravity on a given object. The EP was used by Einstein to derive eqs. (2-3), for example, as discussed in Sect. II. It is easy to see that the equivalence is not perfect, however. For example, the speed of light does not change when the source is accelerated, in accordance with the light-speed constancy postulate of STR. However, it scales as S in the presence of a gravitational field. It is therefore possible to distinguish between kinematic acceleration and gravity on this basis.

The application of the EP for the transverse Doppler effect [21-23] also can be criticized on the basis of the above conversion factors. The argument is made by the various authors that the blue shift observed on the basis of the empirical formula of eq. (7) when the x-ray absorber is at the rim of the rotor arises because the absorber is at a lower gravitational potential in this experimental arrangement. This assumption is in agreement with the gravitational conversion factor for frequency (S) since $\Delta\Phi>0$ and S>1 in this case. Yet, the same argument implies that the total energy of the light source is also greater when it is located near the rotor axis. This conclusion is not correct since the kinetic energy of the source is clearly lower from the vantage point of the absorber since the former is moving at a lower speed. In reality, the entire experiment is carried out at the fixed gravitational potential of the laboratory, so only the Q conversion factor is relevant in the above determinations. Since Q<1 from the vantage point of the absorber [see eq. (13), with v_p being the speed of the light source and v_O being that of the absorber], one arrives at the correct conclusion by simply noting that frequency scales as Q^{-1} while energy scales as Q. The gravitational scale factor is S=1in this case and thus gravity has no influence whatsoever on the observed results. The exponent of Q is different for energy and frequency, whereas that for S is the same, hence the blue shift and the lower energy of the light source relative to the absorber caused by the exclusively kinetic scaling.

One needs both types of conversion factors to describe an object in free fall. The process is governed by the conservation of energy principle. Both scale factors change as the object falls. Let us assume that the object starts at location X with speed v_X relative to its ORS (the ECM) and ends up with speed v_Y at the lower gravitational potential at location Y. The gravitational scale factor has an initial value of S, but is reduced by a factor of $\kappa = A_Y/A_X >1$ (since $r_Y<r_X$) when the free fall is completed [see eqs. (5, 6) for definitions]. Therefore, the final value of the scale factor is S/κ. The energy of the object scales as QS, however, and this total conversion factor must remain constant because of the conservation principle. Consequently, the kinetic scale factor must increase from its initial value of Q when the object is located at X to the larger value of κQ when it reaches Y. Therefore, the following general equation applies based on the definition of Q in eq. (13):

$$\kappa = \gamma(v_Y)/ \gamma(v_X)=A_Y/A_X. \quad (17)$$

This relation is independent of the observer's state of motion because the initial value of Q according to eq. (13) is $\gamma(v_X)/ \gamma(v_O)$, where v_O is the speed of a given observer relative to the ORS, and the final value is $\kappa Q= \gamma(v_Y)/ \gamma(v_O)$. Note that eq. (17) determines the value of v_Y for any given initial value v_X of the object's speed when it is located at X. If the object drops by only a small distance $h=r_X-r_Y$, according to eq. (13) with active mass M,

$$\kappa \approx 1+GMr_Y^{-1}c^{-2}- GMr_X^{-1}c^{-2}= 1+GMc^{-2}(r_Xr_Y)^{-1}(r_X-r_Y) \approx 1 + ghc^{-2}. \quad (18)$$

At the same time, using the definition of the kinetic scale factor Q,

$$\kappa = \gamma(v_Y)/ \gamma(v_X)\approx 1 + 0.5c^{-2} (v_Y^2-v_X^2). \quad (19)$$

Equating these two values of κ then gives the familiar result from classical gravitation theory:

$$v_Y^2= v_X^2 +2gh. \quad (20)$$

In a numerical relativistic calculation of the orbit of Mercury [19], it proved possible to use eq. (17) to compute the final value of the A_i factor at the end of each computational cycle from the initial and final values of the speed of the planet relative to the sun in combination with the value of the factor at the beginning of the cycle. The result obtained for the precession of the perihelion of Mercury and other planets is of the same level of accuracy obtained originally by Einstein based on the general theory of relativity [4], thereby verifying the relation in eq. (17) as well as the use of conversion factors of both kinds to describe the motion of objects in free fall.

CONCLUSIONS

Experiment has shown that the physical properties of objects vary with both their state of motion and their position in a gravitational field. This fact was concealed by the fact that local observers do not notice these changes because they occur uniformly for all objects that are stationary in a given rest frame and gravitational potential. There are no absolutes. Everything is relative. Einstein used his Equivalence Principle (EP) to predict that the frequencies of clocks would increase by a definite factor when they are raised to a higher potential. A useful way to think about this variation of properties is to assume that there is a different set of physical units' operative in each rest frame. Measured values are always expressed in terms of the local set of units, so the reason the observer at a lower potential measures a higher value of frequency than his colleague is simply because his unit of frequency is lower. An important principle of measurement is relevant in this discussion, namely that the numerical value of a property is inversely proportional to the unit in which it is expressed. In order to make sense out of the measured results of different observers, it is necessary to know the values of the conversion factors between their units for each property, the same as if one needs to adjust the value of a distance from meters to feet or a weight from pounds to kilograms in our everyday experience. It is also necessary to know that the values of conversion factors in one direction (feet to meters) are simply the reciprocals of the respective values in the other direction (meters to feet).

It is therefore important to be able to compute the values of the conversion factors for different properties between rest frames. It is shown in the present work that this goal can be accomplished by first determining the values of two quantities, referred to as S and Q above. In the first case, one needs to know the respective positions of the observer and object in the gravitational field in which they are located. The A_i quantities defined in eq. (5) can be determined on this basis for both rest frames and their ratio in eq. (6) is found to be the key factor S. The reverse factor is simply the reciprocal of the latter ratio. The analysis of various experiments shows that the actual conversion factors for the various physical properties are always integral powers of S and the corresponding exponents can be easily determined on the basis of the composition of each property in terms of the fundamental units of time, distance and inertial mass.

The analogous factor for kinetic motion (Q) can always be determined on the basis of information about the speeds of the observer and object relative to a specific rest frame referred to as the ORS [29]. The desired result is given in eq. (13) as a ratio of γ factors. The actual conversion factors for a given property are always integral powers of the fundamental quantity Q, and the pertinent exponents can again be determined exclusively on the basis of the composition of the property in terms of the fundamental units. For the motion of aircraft moving in the gravitational field of the earth, the ECM plays the role of the ORS, whereas in other cases, it can be the axis of a rotor of the rest frame in which a force is applied to the object.

The underlying principle behind the use of systems of physical units and their conversion factors is the objectivity of the measurement process. Observers always see the same events and this makes it possible to be in fundamental agreement with their respective measured results. This condition is not satisfied by Einstein's STR [1] because it holds that measurement is a matter of the perspective of the observer. It is subjective, not objective. Einstein's theory is based on a symmetry principle which holds that two clocks in different inertial systems can be running slower than each other at the same time (symmetric time dilation). This conclusion is based on the Lorentz transformation (LT) and on its condition of Lorentz invariance shown in eq. (17). Experimental studies of the transverse Doppler effect [21-23] and with atomic clocks carried onboard circumnavigating aircraft [24, 25] have contradicted this prediction of STR by showing not only that time dilation is asymmetric, but the rates of clocks are strictly proportional to one another. It is possible to express all of the experimental results in terms of a single relationship, the Universal Time-Dilation Law of eq. (14). Accordingly, it is always possible in principle to say which of two clocks is running slower at a given time, or which distance is longer or which mass is greater. Clock-rate proportionality for inertial clocks is also expected from Newton's First Law since they should all run at constant rates in the absence of any unbalanced external forces.

The conventional view of physicists [31] has been to claim that such experimental data only show that the LT is inapplicable in these cases because either the object of the measurement or the observer is not in uniform translation. However, there is another possibility of bringing relativity theory into agreement with experiment, namely to derive a different space-time transformation that not only is consistent with Einstein's two postulates of relativity and the relativistic space-time transformation (RVT), but also assumes that the measurements of elapsed time by the two observers are strictly proportional to one another: $\Delta t'=\Delta t/Q$. The resulting transformation is shown in eqs. (15a-d) and is referred to as the GPS-LT. Unlike the LT, it is consistent with asymmetric time dilation and clock-rate proportionality and therefore fits in perfectly with the concept of physical units and conversion factors discussed above.

Kinetic and gravitational scaling has proven to be quite important in the operation of the Global Positioning System (GPS). The rates of atomic clocks to be deployed on satellites are pre-corrected [35] prior to launch so that they will be equal to those of clocks located on the earth's surface. This procedure assumes that the clocks will have a constant rate once they reach orbiting speed, which is strictly only true for perfectly circular orbits. In earlier work [13, 36], a method has been described which improves on this approach, at least in principle. The basic idea is to adjust the rates of the clocks continuously based on knowledge of their speeds and positions relative to the ECM. At each time step, the above data can be used to compute instantaneously accurate values of the Q and S factors, which can then be used to make a numerical correction to the time read on the satellite clock. More generally, it is stimulating to imagine that the Q and S factors can be computed with a rather small amount of information for objects located at any point in the universe. They can then be used to convert the results of all physical properties measured locally to the corresponding values in the prevailing system of units on the earth's surface or at some other point in space where such results are required.

The computation and application of the Q and S conversion factors stands in some sense in competition with the General Theory of Relativity [4]. Schiff [18] showed that the "the full structure" of Einstein's theory is not required to quantitatively describe the displacement of star images during solar eclipses, and also that the prediction of the gravitational red shift can be obtained with simpler means as well. In the meantime, it has been shown [19] that the other of the three "crucial tests" of gravitational theory, the precession of the perihelion of Mercury, can also be described with the same accuracy using scaling techniques analogous to those of Schiff.

A more critical question arises because of the relationship between Einstein's Special and General Relativity (GTR). The fact that the LT of STR is contradicted by the observation of asymmetric time dilation in the Hay et al. [21-23] and Hafele-Keating [24-25] studies needs to be considered as a possible violation of GTR as well. Moreover, what is the relationship between GTR and the GPS-LT, which does agree with the above experiments regarding the symmetry of time dilation. At the same time, it is well to recall the test of GTR proposed by Schiff [37, 38]. According to Newton's gravitational theory, the precession of the rate of the component of the spin in the plane of a satellite's orbit should be expected to have a positive sense. Schiff's calculation [38] using GTR indicates on the other hand that the precession frequency should be three times larger than the above value, and most interestingly, have the opposite sense. Gravitational scaling shows only that the value of the frequency should change by a factor of

$S^{-1} > 0$ with the satellite's altitude and would therefore always have a positive sense. Schiff's proposed experiment thus provides a means of differentiating between GTR and gravitational scaling and thus deciding in a definitive manner which one is in better agreement with experiment.

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